An Hamiltonian Approach To State Constrained Optimal Control Problems

Laura Poggiolini*, Francesca C. Chittaro†

We establish sufficient optimality conditions for strong-local optimality of Pontryagin extremals for some single-input control-affine problems. More precisely, we consider the control problem

$$\begin{aligned}
&\dot{\xi}(t) = f_0(\xi(t)) + u(t) f_1(\xi(t)) \quad \text{a.e. } t \in [0, T], \\
&\xi(0) = x_0, \quad \xi(T) \in \mathcal{N}_f, \\
&c(\xi(t)) \le 0 \quad \forall t \in [0, T], \qquad |u(t)| \le 1 \quad \text{a.e. } t \in [0, T],
\end{aligned}$$

where the state space is a smooth manifold M, the function $c \colon M \to \mathbb{R}$ defining the state constraint is assumed to be smooth on a neighborhood of its zero-level set; f_0, f_1 are smooth vector fields on M and $\mathcal{N}_f$ is a submanifold of M. We associate with the problem above a cost J to be minimized, that can be either in Mayer form or the minimum time to reach $\mathcal{N}_f$, i.e. we deal with both the following problems

$$\text{minimize} \quad \psi(\xi(T)), \quad T > 0 \text{ fixed},$$

$$\text{minimize} \quad T, \quad T > 0 \text{ free}.$$

We study extremals containing a bang arc, a boundary arc, followed by a finite number of bang arcs. The sufficient conditions are given by some regularity conditions on the boundary arc, together with a strengthened version of the necessary conditions, and the coerciveness of a suitable finite-dimensional quadratic form. The sufficiency of the provided conditions is proven via Hamiltonian methods. We are currently studying the case when also an internal singular arc is present.

*Dipartimento di Matematica e Informatica, Università degli Studi di Firenze (Italy), laura.poggiolini@unifi.it

†LIS UMR CNRS 7020, Université de Toulon (France), francesca.chittaro@univ-tln.fr